In the periodic mode of operation, it may be assumed that

$$\frac{d}{dt}\Delta a \ll \frac{\omega_0}{Q}(\beta_0 - 1)\Delta a.$$

Therefore, one can write from (13)

$$\Delta a = Z[\cos\varphi + x\cos(\Delta\omega t + \varphi)] \tag{14}$$

where

$$Z = \frac{E_s}{2A_0(\beta_0 - 1)}.$$

Using (14) and substituting $(1+\Delta a)$ for 'a' one can approximate (6) as

$$\frac{d\varphi}{dt} = \Omega + \frac{rZ^2}{2}(1+x^2) - Km_1[\sin\varphi + x\sin(\Delta\omega t + \varphi)] \tag{15}$$

where

$$m_1 = 1 + \frac{Z^2}{2}(1+x^2).$$

It is assumed that the steady-state solution of (15) is

$$\varphi = \varphi_0 + m\sin(\Delta\omega t + \alpha) \tag{16}$$

where φ_0 is the static (i.e., dc) phase error, m is the phase modulation index, and α is a constant. Considering $x<1$, one can arrive at the following relations:

$$\frac{\Omega}{K} + \frac{rZ^2}{2K}(1+x^2) = m_1\left[J_0(m)\sin\varphi_0 + \frac{\frac{m\Delta\omega}{K}J_1(m)}{m_1 J_0(m)}\right] \tag{17}$$

and

$$\left(\frac{m\Delta\omega}{K}\right)^2 + (2J_1(m)m_1\cos\varphi_0)^2 = (xm_1J_0(m))^2. \tag{18}$$

The value of φ_0 for the synchronization boundary can be taken as $\pm(\pi/2 - m)$. Therefore, relations (17) and (18) are transformed to

$$\frac{\Omega}{K} + \frac{rZ^2}{2K}(1+x^2) = \pm m_1 + \frac{m^2}{2}\left[\frac{\Delta\omega}{K} \mp m_1\right] \tag{19}$$

and

$$m^4 m_1^4\left(1 - \frac{x^2}{16}\right) + m^2\left[\left(\frac{\Delta\omega}{K}\right)^2 + \frac{m_1^2x^2}{2}\right] - m_1^2x^2 = 0. \tag{20}$$

Putting

$$\Delta\omega = \Omega_i - \Omega \qquad Z_0 = \frac{\Omega_i - \Omega}{K}$$

and using the value of m^2 from (20), one can write (19) as

$$\frac{\Omega_i}{K} \mp m_1 + \frac{rZ^2}{2K}(1+x^2) = Z_0 + \frac{1}{4m_1^2}[Z_0 \mp m_1]\left[-\left(Z_0^2 + \frac{m_1^2x^2}{2}\right) + \left(\left(Z_0^2 + \frac{m_1^2x^2}{2}\right) + 4m_1^4x^2\right)^{1/2}\right]. \tag{21}$$

This relation is used to calculate the locking bandwidth of the ISO in the presence of the cochannel signal. Fig. 2(b) depicts the variation of the locking bandwidth of a Gunn oscillator with the strength of the cochannel signal for a given synchronizing power and for two different types of interference detuning. It is observed that when $\Omega_i/K \gg 1$, one obtains an increase in the locking range with the strength of the interference, whereas when Ω_i/K is just greater than unity, one obtains a reduction in the locking range of the oscillator with the strength of the cochannel signal; i.e., simply by changing the detuning of the cochannel signal, one can change the performance characteristics relating to the tracking capability of the ISO.

VI. Conclusion

(i) There is an additional shift in the oscillator frequency when the incoming signal is contaminated with the cochannel signal. This effect will be greater when both the synchronizing signal and the cochannel signal are on the same side of the oscillator frequency.

(ii) Simply by varying the frequency of the cochannel signal, one can increase or decrease the tracking capability of the ISO with the strength of the cochannel signal.

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Accurate Characterization of Microstrip Resonator Open End with New Current Expression in Spectral-Domain Approach

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Abstract **—This paper describes a resonant frequency characterization of microstrip resonators which is suitable for very accurate computer-aided design. First, the convergence behavior of the numerical computation based on the spectral-domain approach is discussed to secure the convergence accuracy. To reduce computation time considerably, which at present may amount to several hours, a new current expression in the spectral-domain approach is proposed. In the process described, the computational results under a good convergence have excellent agreement with those measured after estimation of a suitable dielectric constant.**

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I. Introduction

In microstrip design, it is often observed that physical circuit performance differs from that calculated theoretically. This is mainly because of dispersion and discontinuity effects. Many papers have been published about these effects in terms of both quasi-TEM and full-wave analyses [1]–[3]. Simple equations which take dispersion effects into account are available and give very accurate results in certain cases [1]. Discontinuity effects for open strip ends derived from the electrostatic solution are given in a simple equation [2]. Accurate results derived from a rigorous method, however, are not yet available in a simple expression. Although most recent studies on microstrip resonators involve circular, elliptical, or other types [4], [5], a rectangular microstrip resonator is still the most useful in a practical design.

The accuracy required in stripline circuit design depends on the performance required for the MIC subsystem. Designers need to realize better MIC subsystems by precise circuit design as well as by using advanced GaAs devices. To achieve such a goal, computer-aided design (CAD) is commonly used whenever accurate stripline characterizations are indispensable, especially for stripline filter design. As an example, in a parallel coupled microstrip line filter, an accuracy of 0.5 percent is required for the center frequency at 14 GHz. This corresponds to a dimensional accuracy of 20 μm for the length of a line on a 1/40 in. thick alumina substrate.

It is impossible to design such a filter without proper edge effect corrections, which are generally obtained from experimental results in an actual design, even though the experiment is time-consuming. One of the reasons experimental corrections are more reliable than numerically computed data is that most papers do not give information on how well the computed results converge.

Since the open end characterization is a three-dimensional problem, it takes a significant amount of computation time to obtain rigorously well converged results, even if the problem is reduced to a two-dimensional one by a conversion or a transformation. It sometimes takes several hours, for instance, with a VAX-11/780 with a floating point accelerator to obtain results with an error of less than 0.5 percent.

The objective of this paper is to present a quickly converging calculation for the resonant frequency of a microstrip resonator in a short computation time. The analysis is based on the spectral-domain approach (SDA) in which the currents on the resonator are expanded in a series of basis functions [6]. In this new approach, currents are expanded in basis functions with known coefficients, while coefficients are unknown in the conventional (or full) SDA. The results are compared with those of experiments which are processed in a proper manner.

II. Review of Convergence Behavior in the Full SDA

Fig. 1 shows the structure of the microstrip line resonator to be analyzed along with the dimensional parameters. The currents on the strip are expressed in terms of a set of known orthogonal basis functions with unknown coefficients c_μ, d_μ:

$$Jz(x,z)=\sum_{\mu}^{Mz} c_\mu Jz_\mu(x,z) \tag{1}$$

$$Jx(x,z)=\sum_{\mu}^{Mx} d_\mu Jx_\mu(x,z). \tag{2}$$

The basis functions Jz_μ and Jx_μ are separable in the x and z directional variables, so the next equations may be used if a magnetic wall is assumed at $x=0$ and an electric wall at $z=0$:

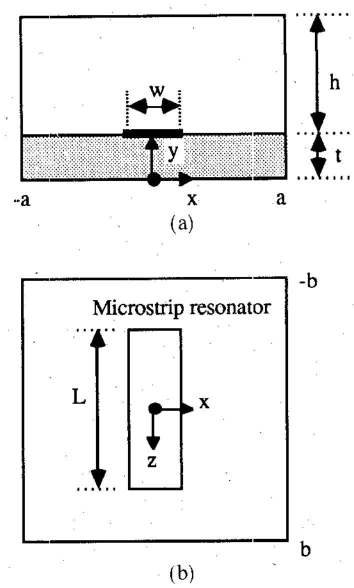

Fig. 1. Structure of microstrip resonator. (a) Cross-sectional side view. (b) Top view.

$$Jz_\mu = Jz_{xr}(x)\cdot Jz_{zs}(z)=\frac{\cos\dfrac{(r-1)\pi x}{w/2}}{\sqrt{1-\left(\dfrac{x}{w/2}\right)^2}}\cdot\cos\frac{(2s-1)\pi z}{L},$$

$$r=1,2\cdots;\ s=1,2\cdots \tag{3}$$

$$Jx_\mu = Jx_{xr}(x)\cdot Jx_{zs}(z)=\frac{\sin\dfrac{r\pi x}{w/2}}{\sqrt{1-\left(\dfrac{x}{w/2}\right)^2}}\cdot\sin\frac{(2s-1)\pi z}{L},$$

$$r=1,2\cdots;\ s=1,2\cdots \tag{4}$$

where r and s are to be chosen on the basis of the number μ. Jz_{xr}, being an x-dependent term, already incorporates the singular behavior of the magnetic fields normal to the edges of the strip.

First, the convergence with respect to the number r was tested by computing the propagation constant β [7] of a conventional 50 Ω microstrip line on a 0.6-mm-thick alumina substrate where the basis functions were assumed as the expansion forms of Jz_{xr} and Jx_{xr}. The results showed that the change in β when r varied from 1 to 6 was less than 0.02 percent. Convergences were also tested for some other structural parameters and it was concluded that, for β calculations of the stripline commonly used in MIC, only the first term is necessary to obtain a good convergence. Thus, the computations in this paper were performed with one x-dependent term for each current.

Next, the convergence in terms of s was tested by computing the resonant frequency of the stripline resonator with a certain length. Fig. 2 shows the results and it is learned that 20 terms could be used satisfactorily to represent the z dependence of the currents with a convergence accuracy of less than 0.3 percent in this case. In the SDA, where the determinant of the matrix is finally computed to solve the eigenvalue problem, each element of the matrix is the inner product of the basis functions and the immittance Green's function [7]. If the number of expansion

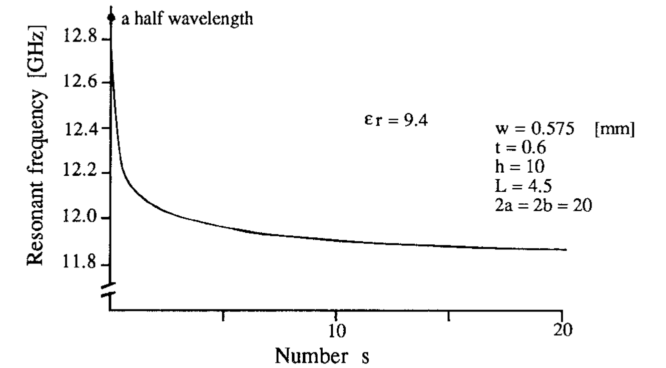

Fig. 2. Convergence of resonant frequency of microstrip resonator by SDA.

terms (basis functions) for $Jz_z(z)$ is N, the same number is required for $Jx_z(z)$ in order to obtain reasonable results, and the matrix size is then $2N \times 2N$. The computation time, therefore, becomes considerably longer as the number N increases. For instance, it takes several hours to obtain one resonant frequency for $N = 20$ using a VAX-11/780.

III. New Current Representation

A new process for reducing the computation time is described where current functions expressed as a series of basis functions with known coefficients are used. Initially, a so-called standard resonator is calculated with the full SDA. Once the resonant frequency is obtained, the coefficients can be calculated from the matrix. Next, a resonator differing from the standard one is calculated, keeping the current coefficients c_μ and d_μ unchanged but varying the evaluated values for Green's functions and the current expansion functions using new structural parameters. As far as the new parameters approximate those of the standard resonator, the results are expected to be accurate because of the variational nature of the Galerkin method in the SDA. This is proved to be true and the results are found to be very practical from a comparison with the results by the full SDA and the experimental results. With this idea, each current is represented in the form of one equation and the matrix size is 2×2. Once the current expansion functions to be used are fixed, the numerical procedure in the SDA requires only that the assumed resonant frequency change until the determinant of the matrix becomes zero. Thus, the computation time can be reduced by a factor of $20 \sim 50$.

Table I shows the calculated results of resonator lengths obtained by the current representation in this new manner in comparison with those obtained by the conventional current expression. In each case from (a) to (d) in Table I, the current coefficients obtained from the case in (a) at 11 GHz were used. These currents are referred to the standard currents and their density shapes are shown in Fig. 3. The dominating longitudinal current is of a perturbed cosine form while the transverse current exists for the most part near the edges and seems to become infinite at the edges. From Table I, it is clear that the results using the new process are in very good agreement with those calculated by conventional means despite the fact that some parameters are not necessarily close to the original ones. Thus, it is illustrated that the numerical computation based on the SDA in this new manner is very practical.

IV. Experimental Results and Discussions

In this section, the accuracy of computational results obtained by the SDA is discussed in relation to the experiments. All computations were done using the new current expression. The experiments were done with a 0.6-mm-thick alumina substrate whose relative dielectric constant ϵ_r as supplied by the manufacturer is 9.4. The accuracy of the value, usually measured at a low frequency such as 1 MHz, is estimated to be ± 2 percent.

TABLE I
Comparison of the Calculated Results of Resonator Lengths by Full SDA and New Process

L in [mm] $\Delta [\%] = \frac{B-A}{A} \times 100$

A : Full SDA
B : New process

f [GHz]	8	11	13
A	6.947	4.889	4.030
B	6.957	4 889	4.033
Δ [%]	0 14	0	0.07

(a) Resonator length vs Frequency

w [mm]	0.3	0.575	1.1
A	5.085	4.889	4.643
B	5.091	4.889	4.651
Δ [%]	0.12	0	0.17

(b) Resonator length vs Strip width

εr	2.1	9.47	13
A	9.900	4.889	4.128
B	9.936	4.889	4.130
Δ [%]	0.36	0	0.05

(c) Resonator length vs Relative dielectric constant

t [mm]	0.3	0.6	1.0
A	4.942	4.889	4.699
B	4.956	4.889	4.704
Δ [%]	0.28	0	0.11

(d) Resonator length vs Substrate thickness

Standard Parameters
fo = 11GHz
εr = 9.47
w = 0.575
t = 0.6
h = 10
2a = 2b = 20
[mm]

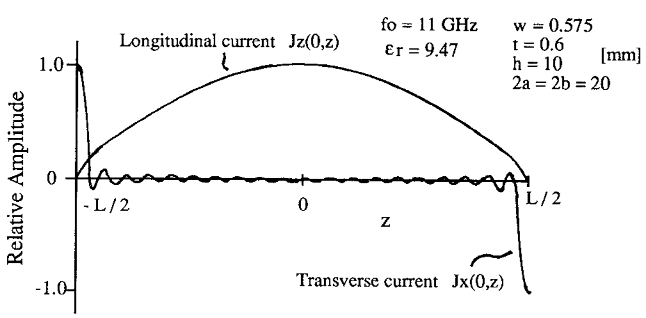

Fig 3 Standard current forms for new process computation.

First, in order to find out the propagation constant β and the open edge effect experimentally, the resonant frequencies of the stripline resonators shown in Fig. 4 were measured. In Fig. 4, the short stripline resonates in the fundamental mode (half-wavelength), and the long one resonates in the second harmonic (full wavelength). It is assumed that the edge effect Δl and the propagation constant β are the same in both resonators when the lengths L_1 and L_2 are chosen so that their resonant frequencies are almost equal. Since two equations are obtained for two unknowns, these can be solved. The relations among these parameters are

$$f_0 = \frac{n \cdot 149.9k}{L_n + 2\Delta l} \tag{5}$$

where $n = 1,2$; k is the relative propagation constant; f_0 is the resonant frequency (in GHz); and L_n is the strip length (in mm).

To examine the accuracy and the uniqueness of the solutions, one more resonator in the third harmonic was tested for different

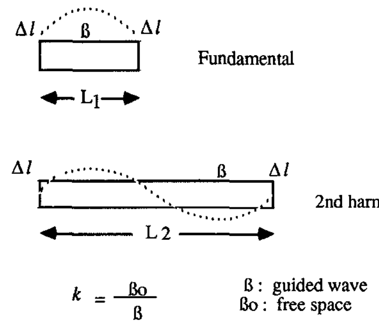

Fig. 4. Stripline resonators for experimental estimation of propagation constant.

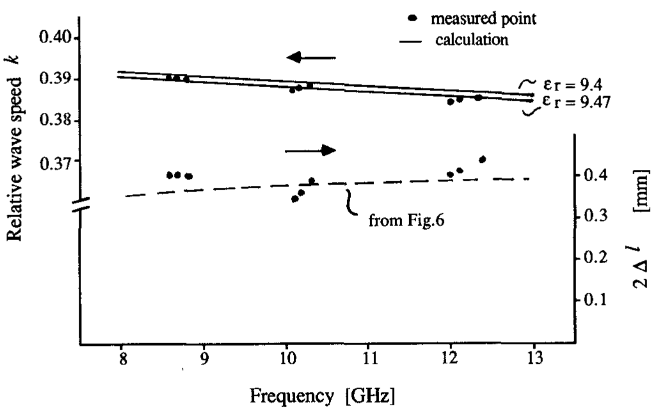

Fig. 5. Experimental results of propagation constant.

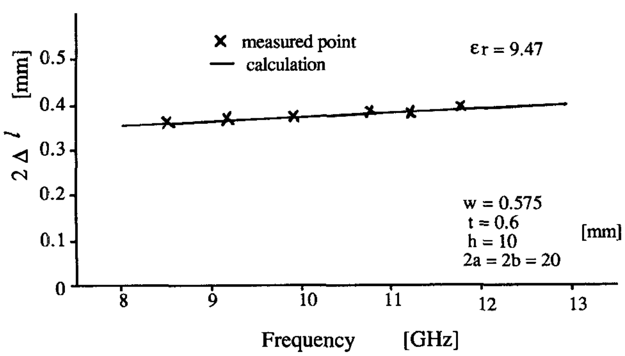

Fig. 6. Open end effect versus frequency.

combinations of equations. Fig. 5 shows the results of k and Δl. The two curves for k show the numerically computed values by the SDA. A better agreement of the curve for $\epsilon_r = 9.47$ with the measured value implies that the originally given ϵ_r value is not necessarily accurate. The measured data for the edge effect Δl do not correlate perfectly and indicate only that $2\Delta l$ has a value of about 0.4 mm. It seems that the accuracy of the measured parameters was not sufficient for Δl estimation.

Next, the resonant frequencies of the stripline resonators were measured with a stripline length varying from 4.5 to 7 mm in the X-band. The computations were performed with the standard structural parameters, an ϵ_r value of 9.47, and the standard currents. The results are that the discrepancy between the measured and calculated values is less than ± 0.2 percent. Then, Δl was evaluated from the differences between the resonator length and the calculated half propagation wavelength at each frequency. Fig. 6 shows the Δl estimations in terms of frequencies.

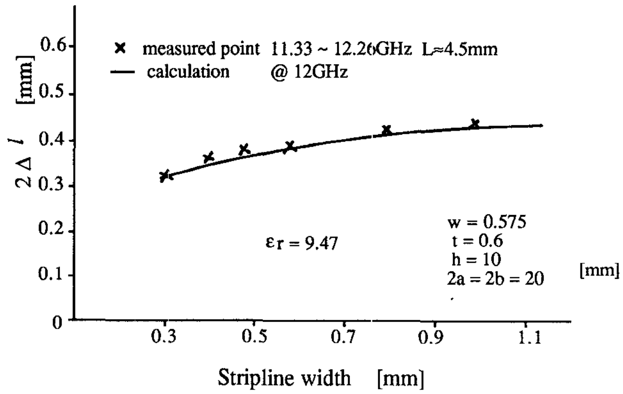

Fig. 7. Open end effect versus stripline width.

The shape of the calculated Δl is also illustrated in Fig. 5 as a dotted line. Evaluations of Δl in terms of strip widths were also carried out. Fig. 7 shows these results. The measured values in the figure were estimated using the calculated half wavelength of the propagation at the resonance frequency of 4.5 mm stripline, while the calculated values are those at 12 GHz.

Excellent agreement between the computed and measured results in both propagation constant and resonant frequency with the same ϵ_r value implies that the SDA is a practical method for exact microstrip characterization. All calculated results in Figs. 6 and 7 also show superior agreement with those measured once the exact ϵ_r value is determined. It is therefore suggested that the ϵ_r value of substrate in a certain frequency band be determined in such a way that the calculated results with the ϵ_r under a good convergence condition meet the experimental results of the resonance frequencies of stripline resonators.

V. Conclusion

Accurate characterization of stripline resonator open end was obtained by numerical computation based on the spectral-domain approach with a resonance frequency accuracy of less than 0.5 percent. The numerical process includes a new current expression which permits the computation time to be 10 to 50 times less than at present. For the evaluation of computational results, the way to determine the ϵ_r value of the substrate at a certain frequency band was discussed. The results obtained in this manner are useful in that they create a simple equation of open edge effects for CAD which includes the frequency dependence.

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